1 Transforms

Fourier Transform

$$x_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt$$

Inverse Fourier Transform

$$x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega$$

Fourier Transform of Sampled Signal

$$X_p(j\Omega) = \frac{\Omega_T}{2\pi}\sum_{k=-\infty}^{\infty} x_a(j(\Omega + k\Omega_T))$$

$$\Omega_T = \frac{2\pi}{T}$$

Interpolation

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT)\,\mathrm{sinc}(\frac{t}{T} - n)$$

DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

Inverse DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

DTFT $\leftrightarrow$ FT

$$X(e^{j\omega}) = X_p(j\omega/T) = \frac{1}{T}\sum_{k=-\infty}^{\infty} x_a(j(\omega + 2\pi k)\frac{1}{T}))$$

Z-Transform

The DTFT is the ZT on the unit circle if $z = re^{j\omega}$ converges

$$G(z) = \sum_{n=-\infty}^{\infty} g[n]z^{-n}$$

$$z \in C$$

Inverse Z-Transform

$$g[n] = \frac{1}{2\pi j}\oint_C G(z)z^{n-1}dz$$

Residue

$$\lim_{z\to\lambda_0}(z - \lambda_0)G(z)z^{n-1}$$

$$\frac{1}{m-1}!\lim_{z\to\lambda_0}\left\{\frac{d^{m-1}}{d_z^{m-1}}(z - \lambda 0^m G(z)z^{n-1}\right\}$$

ROC

$$\sum_{n=-\infty}^{\infty} |g[n]|r^{-n} < \infty$$

2 Trigonometry

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin(\alpha)\cos(\beta) = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$$

$$\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

3 Analog Filters

	Equiripple	Monotonic
Chebyshev Type I	PB	SB
Chebyshev Type II	SB	PB
Elliptic	PB, SB	

Ideal LPF

$$H(s) = \frac{\Omega_1}{s + \Omega_1}$$

$$H(j\Omega) = \frac{\Omega_1}{j\Omega + \Omega_1}$$

3.1 Spectral Transforms

$$\Omega_0 = \hat{\Omega}_0^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2}$$

$$\mathrm{BW} = \hat{\Omega}_{p2} - \hat{\Omega}_{p1}$$

Analog Highpass

$$s = \frac{\Omega_p\hat{\Omega}_p}{\hat{s}}$$

$$\Omega = \frac{\Omega_p\hat{\Omega}_p}{\hat{\Omega}}$$

Analog Bandpass

$$s = \Omega_p\frac{\hat{s}^2 + \hat{\Omega}_{p1}\hat{\Omega}_{p2}}{\hat{s}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

$$\Omega = -\Omega_p\frac{\hat{\Omega}_{p1}\hat{\Omega}_{p2} - \hat{\Omega}^2}{\hat{\Omega}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

Analog Bandstop

$$s = \frac{\Omega_s\hat{s}(\hat{\Omega}_{s2} - \hat{\Omega}_{s1})}{\hat{s}^2 + \hat{\Omega}_{s1}\hat{\Omega}_{s2}}$$

$$\Omega = \frac{\Omega_s \hat{\Omega}(\hat{\Omega}_{s2} - \hat{\Omega}_{s1})}{\hat{\Omega}_{s1}\hat{\Omega}_{s2} - \hat{\Omega}^2}$$

Ripple Specifications

$$1 - \delta_p \le |H_a(j\omega)| \le 1 + \delta_p$$

$$\alpha_p = -20\log_{10}(1 - \delta_p)$$

$$|H_a(j\Omega)| < \delta_s$$

$$\alpha_s = -20\log_{10}(\delta_s)$$

For no anti-aliasing in the interesting band

$$\Omega_s < \Omega_T - \Omega_p$$

Bilinear Transform

LHP maps to inside of UC

Im axis maps to the UC

$$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$$

$$z = \frac{1 + (T/2)s}{1 - (T/2)s}$$

$$\Omega = \frac{2}{T}\tan(\omega/2)$$

$$\omega = 2\arctan(\Omega T/2)$$

3.2 Butterworth LPF

Poles of $H(s)H(-s)$ are equally spaced on $r = \Omega_c$

$$H(s)H(-s) = \frac{1}{1 + (-s^2/\Omega_c^2)^N}$$

$$|H(j\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_C)^{2N}}$$

Minimum passband gain

$$1 - \delta_p = \frac{1}{\sqrt{1 + \epsilon^2}}$$

$$\epsilon^2 = \frac{1}{(1 - \delta_p)^2} - 1$$

Order

$$A = \frac{1}{\delta_s}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

$$k_1 = \epsilon/\sqrt{A^2 - 1}$$

$$N = \frac{\log((1 - \delta_s^2)/(\delta_s^2\epsilon^2))}{2\log(\Omega_s/\Omega_p)} = \frac{\log(1/k_1)}{\log(1/k)} = \frac{\log(\sqrt{A^2 - 1}/\epsilon)}{\log(\Omega_s/\Omega_p)}$$

Determine Ω_c:

Exceed specification in stopband

$$\frac{1}{1 + (\Omega_p/\Omega_c)^{2N}} = \frac{1}{1 + \epsilon^2} = (1 - \delta_p)^2$$

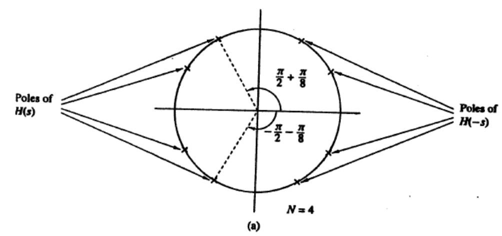
Exceed specification in passband

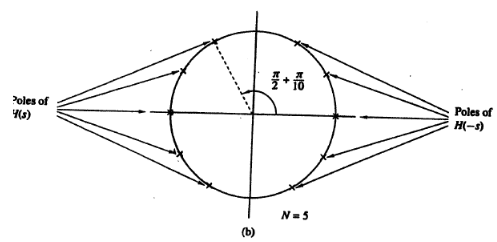
$$\frac{1}{1 + (\Omega_s/\Omega_c)^{2N}} = \delta_s^2 = \frac{1}{A^2}$$

LHP Poles:

$$(-s^2/\Omega_c^2)^N = -1$$

$$s = \Omega_c e^{j\frac{\pi}{2}} e^{j(1+2k)\pi/(2N)} \text{VERIFY}$$





3.3 Chebyshev LPF

Type 1

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \epsilon^2 T_N(\Omega/\Omega_p)}$$

Type 2

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \epsilon^2(\frac{T_N(\Omega_s/\Omega_p)}{T_N(\Omega_s/\Omega)})}$$

$$T_N = \begin{cases} \cos(N\arccos(\Omega)) & |\Omega| \le 1 \\ \cosh(N\,\text{arccosh}(\Omega)) & |\Omega| > 1 \end{cases}$$

3.4 Elliptic

Faster transition band, equiripple everywhere

4 Phase and Delay

Phase Delay (Carrier)

$$\tau_p(\omega_0) = -\frac{\theta(\omega_0)}{\omega_0}$$

Group Delay (Envelope)

If group delay is constant, there is linear phase

$$\tau_g(\omega_0) = -\frac{d\theta(\omega)}{d\omega}|_{\omega=\omega_0}$$

Linear Phase Filter

$$H(e^{j\omega}) = e^{-j\omega D}e^{j\beta}\hat{H}(w)$$

$\hat{H}(w)$ is real

$$\tau_g(\omega) = D$$

4.1 Minimum Phase Transfer Function

Minimum	Zeros all in UC
Non-Minimum	Zeros outside UC
Mixed	Zeros inside/outside UC
Maximum	All zeros outside UC

Can represent any transfer function as

$$H(z) = H_{min}(z)A(z)$$

Where $H_{min}(z)$ is a minimum phase transfer function

$A(z)$ is an all pass filter

$$H_{min}(z) = H(z)\underbrace{\frac{z^{-1}-c_1^*}{1-c_1z^{-1}}\cdots\frac{z^{-1}-c_M^*}{1-c_Mz^{-1}}}_{\text{unit magnitude all-pass filter}}$$

4.2 All Pass Filter

$$|A(e^{j\omega})| = 1 \text{ for all } \omega$$

$$A_M(z) = \pm\frac{d_M + d_{M-1}z^{-1} + \cdots + d_1z^{-M+1} + z^{-M}}{1 + d_1z^{-1} + \cdots + d_{M-1}z^{-M+1} + d_Mz^{-M}}$$

- Poles/Zeros are mirrored
- Unwrapped phase is a decreasing function of frequency
- Group delay is positive for all ω
- Phase change from $\omega = 0 \to \omega = \pi$ is $\int_0^\pi \tau_g(\omega)d\omega = M\pi$

4.3 Zero Phase Transfer Function

Given a filter $H(z)$

$$F(z) = H(z)H(z^{-1})$$

$z = v$ is a pole of $F(z)$ and $z = \frac{1}{v}$ is also a pole

5 FIR Filters

- Stable
- Easy to implement
- Linear phase can be guaranteed

$$y[n] = b_0x[n] + b_1x[n-1] + \cdots + b_Mx[n-M]$$

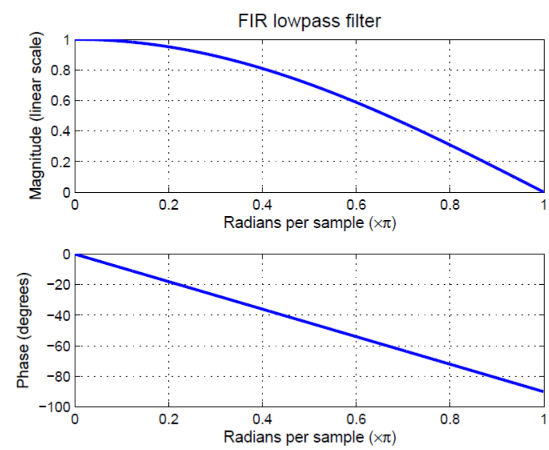
$$H(e^{j\omega}) = b_0 + b_1e^{-j\omega} + \cdots + b_Me^{-j\omega M}$$

Low Pass

$$H(z) = \frac{z+1}{2z}$$

$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$$

$$H(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$$

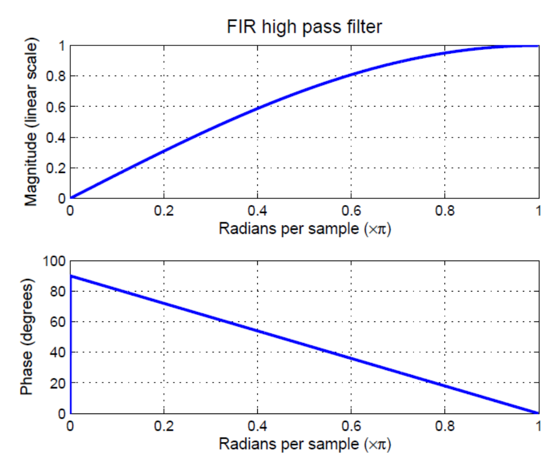


High Pass

$$H(z) = \frac{z-1}{2z}$$

$$y[n] = \frac{1}{2}x[n] - \frac{1}{2}x[n-1]$$

$$H(e^{j\omega}) = e^{-j(\omega/2-\pi/2)}\sin(\omega/2)$$



5.1 Classification

Symmetric

$$H(z) = z^{-N} H(z^{-1})$$

$$h[n] = h[N-n]$$

Antisymmetric

$$H(z) = -z^{-N} H(z^{-1})$$

$$h[n] = -h[N-n]$$

Type I

N - even

$$h[n] = h[N-n]$$

$$\breve{H}(\omega) = h[\frac{N}{2}] + 2\sum_{n=1}^{N/2} h[\frac{N}{2} - n]\cos(\omega n)$$

Type II

Must have a zero at $z = -1$, not suitable for HPF

N - odd

$$h[n] = h[N-n]$$

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\frac{N+1}{2} - n]\cos(\omega(n - \frac{1}{2}))$$

Type III

Must have a zero at $z = -1$ and $z = 1$, not suitable for LPF, HPF

N - even

$$h[n] = -h[N-n]$$

$$\breve{H}(\omega) = 2\sum_{n=1}^{N/2} h[\frac{N}{2} - n]\sin(\omega n)$$

Type IV

Must have a zero at $z = 1$, not suitable for LPF

N - odd

$$h[n] = -h[N-n]$$

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\frac{N+1}{2} - n]\sin(\omega(n - \frac{1}{2}))$$

Linear Phase

For linear phase causal FIR, must contain term $e^{-j\frac{N}{2}\omega}$

Symmetric and Antisymmetric filters are always linear

5.2 Ideal FIR impulse response

$H_e^{j\omega} = 1$ in passband

Low Pass

$$h_{LP}[n] = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\sin w_c n}{\pi n} & n \neq 0 \end{cases}$$

High Pass

$$h_{HP}[n] = \begin{cases} 1 - \frac{\omega_c}{\pi} & n = 0 \\ -\frac{\sin \omega_c n}{\pi n} & n \neq 0 \end{cases}$$

Band Pass

$$h_{BP}[n] = \begin{cases} \frac{\omega_{c2} - \omega_{c1}}{\pi} & n = 0 \\ \frac{\sin \omega_{c2} n}{\pi n} - \frac{\sin \omega_{c1} n}{\pi n} & n \neq 0 \end{cases}$$

Band Stop

$$h_{BS}[n] = \begin{cases} 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi} & n = 0 \\ \frac{\sin \omega_{c1} n}{\pi n} - \frac{\sin \omega_{c2} n}{\pi n} & n \neq 0 \end{cases}$$

5.3 Windowing

- No precise edge frequency control
- Width of the transistion region between passband and stopband in $H(e^{j\omega})$ increases with width of main lobe of $W(e^{j\omega})$
- Ripple in PB/SB is dependent on area under sidelobes
- If N increases the width of the main lobe decreases but area under sidelobes remain constant. This means transistion region smaller but ripple remains.
- Ripple caused by rectangular window is usually not acceptable, and instead a window which tapers smoothly to zero at each end is used.
- Reduced ripple has tradeoff with wider transition region. Can compensate by increasing N.

Properties

All symmetric, so ripple in PB and SB are equal

Window	Width mainlobe	Peak sidelobe	$20log_{10}\delta$
Rectangular	$4\pi/N$	-13dB	-21dB
Hanning	$8\pi/N$	-32dB	-44dB
Hamming	$8\pi/N$	-43dB	-54dB
Blackman	$12\pi/N$	-58dB	-75dB

Hanning

$$w[n] = 0.5 - 0.5\cos(\frac{2\pi n}{N})$$

Hamming

$$w[n] = 0.54 - 0.46\cos(\frac{2\pi n}{N})$$

Blackman

$$w[n] = 0.42 - 0.5\cos(\frac{2\pi n}{N} + 0.08\cos(\frac{4\pi n}{N})$$

5.4 Parks McClellan Method

Minimises weighted peak error in passbands and stopbands (equiripple in both)

$$\epsilon(\omega) = W(e^{j\omega})[H(e^{j\omega}) - D(e^{j\omega})]$$

Least squares criterion

$$\min_{H(e^{j\omega})} \int_{-\pi}^{\pi} |W(e^{j\omega})|[H(e^{j\omega}) - D(e^{j\omega})]|^2 d\omega$$

Minimax / Chebyshev criterion

Where R is a set of disjoint frequency bands comprising the passbands and the stopbands.
Used by the Parks-McClelland method

$$\min_{H(e^{j\omega})} \max_{\omega \in R} |W(e^{j\omega})|[H(e^{j\omega}) - D(e^{j\omega})]|$$

Error function for linear phase filters

Where $\breve{H}(\omega)$ is real

$$H(e^{j\omega}) = e^{-jN\omega/2} e^{j\beta} \breve{H}(\omega)$$

6 IIR Filters (causal)

- Smaller ripples
- Fewer parameters
- Lower computational complexity and memory

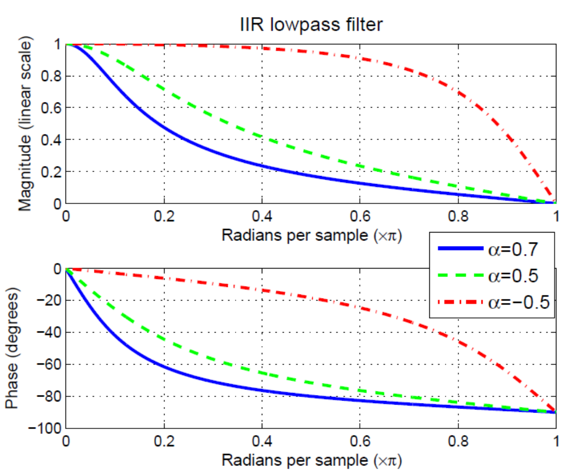
$$y[n]+\alpha_1 y[n-1]+\cdots+\alpha_N y[n-N] = b_0+b_1 e^{-j\omega}+\cdots+b_M e^{-j\omega M}$$

$$H(e^{j\omega}) = \frac{b_0 + b_1 e^{-j\omega} + \cdots + b_M e^{-j\omega M}}{\alpha_1 y[n-1] + \cdots + \alpha_N y[n-N]}$$

Low Pass

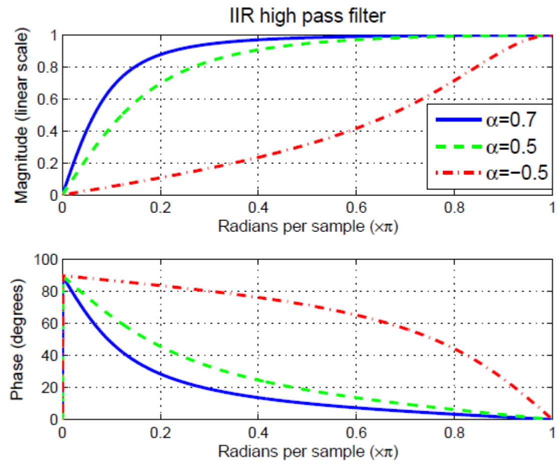
$$H_{LP}(z) = \frac{1-\alpha}{z} \frac{1+z^{-1}}{1-\alpha z^{-1}}$$

$$|H_{LP}(e^{j\omega})|^2 = \frac{(1-\alpha)^2(1+\cos(\omega))}{2(1+\alpha^2 - 2\alpha\cos(\omega))}$$



High Pass

$$H_{HP}(z) = \frac{1+\alpha}{2} \frac{1-z^{-1}}{1-\alpha z^{-1}}$$



Band Pass

$$H_{BP}(z) = \frac{1-\alpha}{2} \frac{1-z^{-2}}{2-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}$$

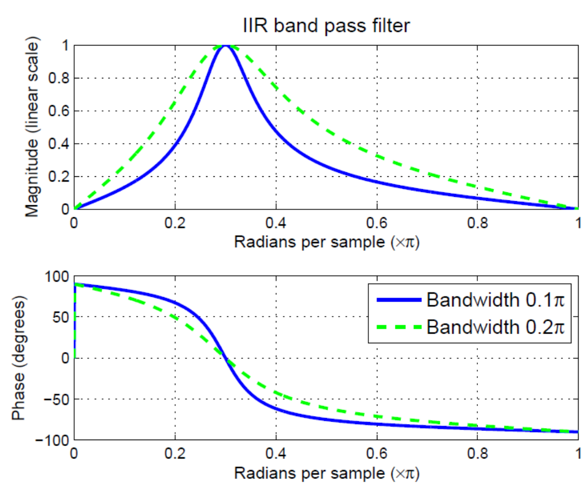
$$r = \sqrt{\alpha}$$

$$\phi = \arccos(\beta(1+\alpha)/(2\sqrt(\alpha))$$

$$|H_{BP}(e^{j\omega})|^2 = \frac{(1-\alpha)^2 \sin^2(\omega)}{(1+\alpha)^2(3-\cos(\omega))^2 + (1-\alpha)^2 \sin^2(w)}$$

$$\omega_0 = \arccos(\beta)$$

$$\text{BW} = \omega_{c2} - \omega_{c1} = \arccos(\frac{2\alpha}{1+\alpha^2})$$

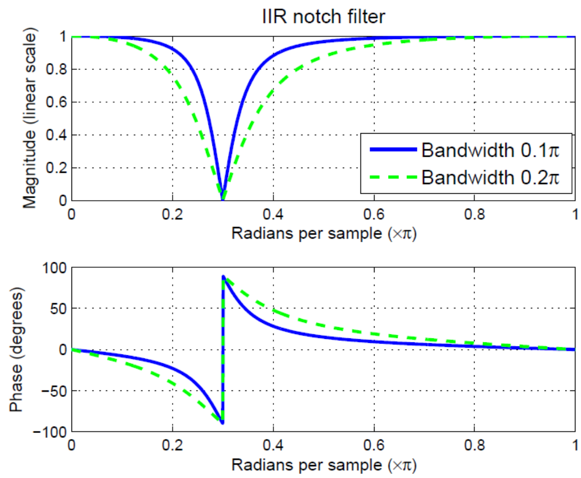


Band Stop

Magnitude is 0 at ω_0 and 1 at $0,\pi$

$$H_{BS}(z) = \frac{1+\alpha}{2} \frac{1-2\beta z^{-1}+z^{-2}}{1+\beta(1+\alpha)z^{-1}+\alpha z^{-2}}$$

$$\text{BW} = \arccos\left(\frac{2\alpha}{1+\alpha^2}\right)$$



7 Causality and Stability

Causality

- $H(e^{j\omega})$ cannot be 0 except at a finite number of ω
- $|H(e^{j\omega}|$ cannot be constant in the frequency band
- $|H(e^{j\omega}|$ and phase $\angle H(e^{j\omega})$ are dependent on each other

Stability

- BIBO stability is defined as (for continuous and discrete time respectively)

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty$$

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

- A transfer function is stable if all the poles are in the unit circle

8 Discrete Fourier Transform

L = Length of data sequence
M = FIR Filter Length
N = Number of frequencies the DFT is sampled at

N-point DFT

$$X[k] = X\left(e^{j\frac{2\pi k}{N}}\right) = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}}$$

$$k = 0, 1, 2, \dots, N-1$$

Inverse DFT

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}} = x[n]$$

$$n = 0, 1, \dots, ..., N-1$$

Condition for recovery

x[n] is time limited to less than N

$$x[n] = \begin{cases} x_p[n] & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

8.1 DFT as a linear transformation

$$W_N = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W_N & W_N^2 & \dots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

DFT

$$X_N = \boldsymbol{W}_N x_N$$

IDFT

$$x_N = \boldsymbol{W}_N^{-1} X_N = \frac{1}{N}\boldsymbol{W}_N^* X_N$$

8.2 Properties

Periodicity

$X[k]$ and $x[n]$ are periodic with period N

Linearity

$$x[n] = a_1x_1[n] + a_2x_2[n]$$

$$X[k] = a_1X_1[k] + a_2X_2[k]$$

Conjugate

$$X^*[k] \leftrightarrow x^*[\langle -k\rangle_N]$$

Time Shift

$$e^{\frac{-j2\pi ka}{N}}X[k] \leftrightarrow x[\langle n-a\rangle_N]$$

Even/Odd

$$x_{even}[n] = \frac{1}{2}(x[n] + x^*[\langle -n\rangle_N]) \leftrightarrow \Re\{X[k]\}$$

$$x_{odd}[n] = \frac{1}{2}(x[n] - x^*[\langle -n\rangle_N]) \leftrightarrow j\Im\{X[k]\}$$

Symmetry

For a real sequence $x[n] = x^*[n]$

$$X[N-k] = X^*[k] = X[\langle -k\rangle_N]$$

For $x[n]$ real and even, $X[k]$ real and even
For $x[n]$ real and odd, $X[k]$ imaginary and odd

Parseval's relation

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$

8.3 Circular Convolution

Can be turned into a linear convolution by padding $x_1[n]$ and $x_2[n]$ with zeros so they have length at least $L+M-1$

$$x_1[n] \circledast x_2[n] \equiv \sum_{k=0}^{N-1} x_1[k]x_2[\langle n-k\rangle_N]$$

Example

$$x_1 = \{1,2,0\}, \quad x_2 = \{3,5,4\}$$

k	-2	-1	0	1	2	3	
$x_1[k]$			1	2	0		
$x_2[\langle -k\rangle_3]$	4	5	3	4	5	3	$11 = y[0]$
$x_2[1\langle -k\rangle_3]$	3	4	5	3	4	5	$11 = y[1]$
$x_2[2\langle -k\rangle_3]$	5	3	4	5	3	4	$14 = y[2]$
$x_2[3\langle -k\rangle_3]$	4	5	3	4	5	3	$11 = y[3]$
$(= x_2[\langle -k\rangle_3]$							

Circular shift

$$\langle m\rangle_N = \begin{cases} \text{rem}(m,N) & m \geq 0 \\ \text{rem}(m,N)+N & m<0, \text{rem}(m,N) \neq 0 \\ 0 & m<0, \text{rem}(m,N)=0 \end{cases}$$

Circular time shift

$$x_1[n] = x[\langle n-l\rangle_N]$$

$$X_1[k] = X[k]e^{-j\frac{2\pi kl}{N}}$$

Circular frequency shift

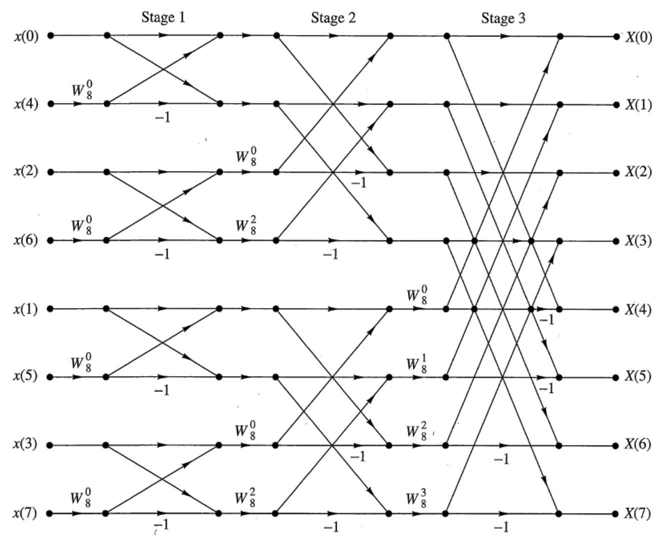
$$x_1[n] = x[n]e^{j\frac{2\pi ln}{N}}$$

$$X_1[k] = X[\langle k-l\rangle_N]$$

8.4 Fast Fourier Transform

- Direct computation of DFT is $O(N^2)$
- FFT algorithm is $O(N\log N)$
- FFT exploits periodicity and symmetry
- Need to zero pad for length $N = 2^\nu$
- $\log_2(N)$ stages, with $N/2$ multiplications at each

FFT Butterfly



Computations per output data point

$$\frac{N\log_2(2N)}{L} = \frac{N\log_2(2N)}{N-M+1}$$

$$c(\nu) \approx \frac{2^\nu(\nu+1)}{2^\nu - M}$$

9 Multi Rate Signal Processing

9.1 Up Sampler

$$x_u[n] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L, \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$X_u(e^{j\omega}) = X(e^{j\omega L})$$

Interpolator

Low pass filter to remove frequency domain images
Gain of LPF should compensate for insertion of zeros, and hence should be L.

$$X_u(e^{j\omega}) = \frac{1}{L}X_1(e^{j\omega})$$

- In the frequency domain the spectrum is compressed with factor L and we get $L-1$ additional images of the spectrum.
- Sampling at $F_s = 1/T$, highest frequency component of $x[n]$ is $1/2T$ Hz corresponding to π radians/sample.
- Highest frequency after upsampling of x_u is π radians/sample corresponding to $L/2T$ Hz.

9.2 Down Sampler

$$y[n] = x[nM]$$

$$Y(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega+2\pi k)/M})$$

Single Stage Decimator

Frequencies above π/M are aliased and should be removed (decimated) prior to down-sampling

$$\begin{aligned} \text{Passband} \quad & 0 \leq F \leq F_p \\ \text{Transition region} \quad & F_p \leq F \leq F_s (\leq F_0/2M) \\ \text{Stopband} \quad & F_s \leq F \leq F_0/2 \end{aligned}$$

Multi stage decimator

A factor M decimator can be implemented in K stages with the advantage of relaxed filter specifications

$$M = \Pi_{i=1}^{K} M_i$$

F_0 : sampling frequency at input of decimator
F_i : sampling frequency at output of ith stage
$F_i = \frac{F_{i-1}}{M_i}$

ith stage:

$$\begin{aligned} \text{Passband} \quad & 0 \leq F \leq F_p \\ \text{Transition region} \quad & F_p \leq F \leq F_i - F_s \\ \text{Stopband} \quad & F_i - F_s \leq F \leq F_{i-1}/2 \end{aligned}$$

Last stage:

$$\begin{aligned} \text{Passband} \quad & 0 \leq F \leq F_p \\ \text{Transition region} \quad & F_p \leq F \leq F_s \\ \text{Stopband} \quad & F_s \leq F \leq F_{K-1}/2 \end{aligned}$$

Ripple Specification (Two stage example)

$$\begin{aligned} \text{Passband} \quad & \delta_p^{'} = \sqrt{1+\delta_p} - 1 \\ \text{Stopband} \quad & \delta_s^{'} = \delta_s \end{aligned}$$

Decimation filter computational saving

$$\frac{(2K+1)M}{KM+K+1}$$

10 Energy Spectrum

Energy over interval N

$$E_N = \sum_{n=-N}^{N} |x[n]|^2$$

Energy Signals

$$\begin{aligned} E &= \lim_{N\to\infty} E_N \\ &= \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty \end{aligned}$$

Power Signals

$$\begin{aligned} P &= \lim_{N\to\infty} \frac{1}{2N+1} E_N \\ &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 < \infty \end{aligned}$$

Crosscorrelation

Measure of the degree to which two signals resemble each other

$$r_{xy}[l] = \sum_{n=-\infty}^{\infty} x[n]y[n-l], \quad l = 0, \pm 1, \pm 2, \ldots$$

$$r_{xy}[l] = r_{yx}[-l]$$

$$|r_{xy}[l]| \leq \sqrt{r_{xx}[0] r_{yy}[0]}$$

Autocorrelation

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l], \quad l = 0, \pm 1, \pm 2, \ldots$$

$$r_{xx}[l] = r_{xx}[-l]$$

$$|r_{xx}[l]| \leq r_{xx}[0]$$

Wiener-Khintchine theorem:

$S_{xx}(e^{j\omega})$ is the DTFT of $r_{xx}[n]$

Implies two ways of computing energy spectrum

1. Compute the DTFT $X(e^{j\omega})$ and $S_{xx}(e^{j\omega}) = |X(e^{j\omega})|^2$
2. Compute the autocorrelation $r_{xx}[l]$ and $S_{xx}(e^{j\omega}) = \text{DTFT}\{r_{xx}[l]\}$

10.1 Estimation of energy spectrum

Direct method

$$S_{xx}(e^{j\omega}) = |X(e^{j\omega})|^2 = |\sum_{n=-\infty}^{\infty} x[n] e^{j\omega n}|^2$$

Estimate of energy spectrum

$$S_{\tilde{x}\tilde{x}}(e^{j\omega}) = |\sum_{n=0}^{N-1} x[n] e^{j\omega n}|^2$$

Windowing

$$\tilde{x}[n] = w[n]x[n]$$

- Convolution with mainlobe smooths the estimate over nearby frequencies
- The frequency resolution is determined by the width of the mainlobe
- The sidelobes cause sidelobe energy to appear in the spectrum. This is called spectral leakage
- Smoothing caused by window can be a problem when we need to resolve signals with closely spaced frequency components

Rectangular window

High frequency resolution but large spectral leakage

Hamming window

Lower frequency resolution but smaller spectral leakage

Spectrum estimation with DFT

$$S_{\tilde{x}\tilde{x}}(e^{j\frac{2\pi k}{N}}) = |\sum_{n=0}^{N-1} x[n]e^{j\frac{2\pi k}{N}n}|^2$$

If a clearer picture is needed the data sequence can be padded with zeros to obtain values at more frequencies (without increasing resolution).
Resolution can be increased by using more data points

11 Useful Properties

$$\frac{dX(e^{j\omega})}{d\omega} = \frac{d}{d\omega}\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega} = \sum_{n=-\infty}^{\infty} -jnx[n]e^{-j\omega n}$$

Complex exponential shorthand

$$W_M^N = e^{-j2\pi N/M}$$

Parseval's Theorem

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

Energy Density Spectrum

$$\delta_{xx}(e^{j\omega}) = |X(e^{j\omega})|^2 = X(e^{j\omega})X(e^{-j\omega})$$

Convolution

$$x[n] = x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

Modulation

$$x[n] = x_1[n]\cos(\omega_0 n)$$

$$X(e^{j\omega}) = \frac{1}{2}(X_1(e^{j(\omega+\omega_0)}) + X_1(e^{j(\omega-\omega_0)})$$

Geometric Series

$$\sum_{k=0}^{\infty} r_k = \frac{1}{1-r}$$

$$\sum_{k=1}^{\infty} r_k = \frac{r}{1-r}$$

$$\sum_{n=0}^{\infty}(n+1)x^n = \frac{1}{(1-x)^2}$$

Obtaining an even/odd sequence

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Frequency Response from Pole Zero Plot

$$|H(e^{j\omega})| = \frac{\Pi_{\text{zeroes}}\text{Vector length from zero to } \omega}{\Pi_{\text{poles}}\text{Vector length from pole to } \omega}$$

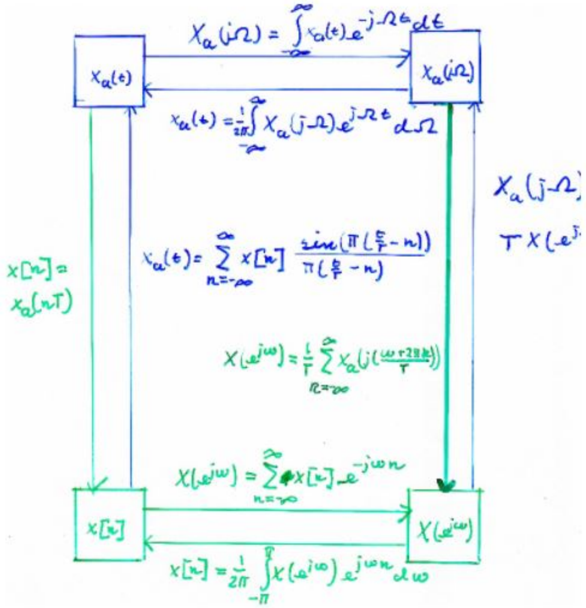
$$\angle H(e^{j\omega}) = \sum_{\text{zeroes}} \angle z_i - \sum_{\text{poles}} \angle p_i$$

Integration by parts

$$\int udv = uv - \int vdu$$

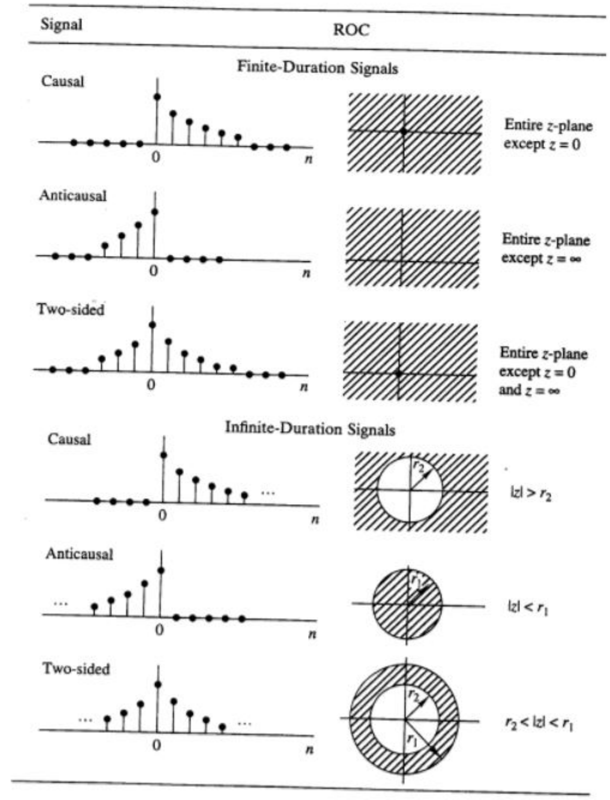
Partial fraction decomposition table

Type	Factor example	Decomposition
Linear factor	$(x-4)$	$\frac{A}{x-4}$
Repeated linear factor	$(x-4)^2$	$\frac{A}{(x-4)}+\frac{B}{(x-4)^2}$
Quadratic irreducible factor	(x^2+4)	$\frac{Ax+B}{(x^2+4)}$
Repeated quadratic irreducible factor	$(x^2+4)^2$	$\frac{Ax+B}{(x^2+4)}+\frac{Cx+D}{(x^2+4)^2}$



Sec. 3.1 The z-Transform

TABLE 3.1 CHARACTERISTIC FAMILIES OF SIGNALS WITH THEIR CORRESPONDING ROC



The z-Transform and Its Application to the Analysis of LTI Systems

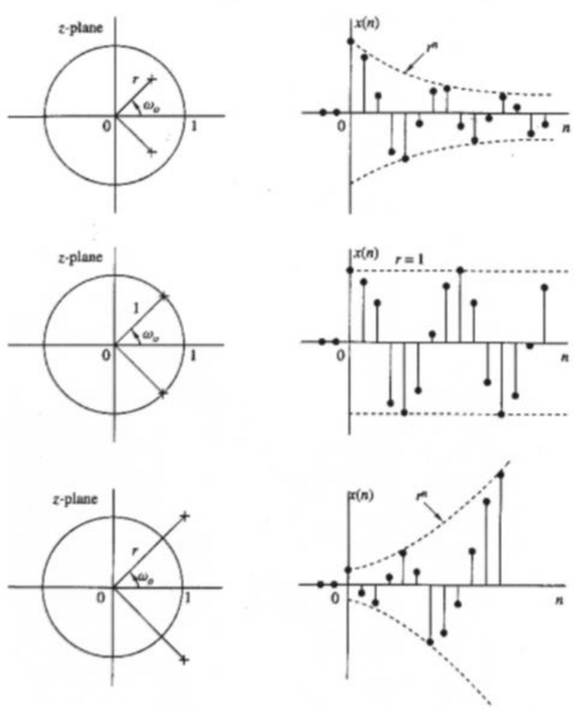


Figure 3.13 A pair of complex-conjugate poles corresponds to causal signals with oscillatory behavior.

Figure 9.1 in Mitra. Spectral transformations in the discrete domain.

Table 9.1: Spectral transformations of a lowpass filter with a cutoff frequency ω_c.

Filter type	Spectral Transformation	Design Parameters
Lowpass	$z^{-1}=\frac{\hat{z}^{-1}-\lambda}{1-\lambda\hat{z}^{-1}}$	$\lambda=\frac{\sin\left(\frac{\omega_c-\hat{\omega}_c}{2}\right)}{\sin\left(\frac{\omega_c+\hat{\omega}_c}{2}\right)}$ $\hat{\omega}_c=$ desired cutoff frequency
Highpass	$z^{-1}=-\frac{\hat{z}^{-1}+\lambda}{1+\lambda\hat{z}^{-1}}$	$\lambda=-\frac{\cos\left(\frac{\omega_c+\hat{\omega}_c}{2}\right)}{\cos\left(\frac{\omega_c-\hat{\omega}_c}{2}\right)}$ $\hat{\omega}_c=$ desired cutoff frequency
Bandpass	$z^{-1}=-\frac{\hat{z}^{-2}-\frac{2\lambda\rho}{\rho+1}\hat{z}^{-1}+\frac{\rho-1}{\rho+1}}{\frac{\rho-1}{\rho+1}\hat{z}^{-2}-\frac{2\lambda\rho}{\rho+1}\hat{z}^{-1}+1}$	$\lambda=\frac{\cos\left(\frac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\frac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)}$ $\rho=\cot\left(\frac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)\tan\left(\frac{\omega_c}{2}\right)$ $\hat{\omega}_{c2},\hat{\omega}_{c1}=$ desired upper and lower cutoff frequencies
Bandstop	$z^{-1}=\frac{\hat{z}^{-2}-\frac{2\lambda}{1+\rho}\hat{z}^{-1}+\frac{1-\rho}{1+\rho}}{\frac{1-\rho}{1+\rho}\hat{z}^{-2}-\frac{2\lambda}{1+\rho}\hat{z}^{-1}+1}$	$\lambda=\frac{\cos\left(\frac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\frac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)}$ $\rho=\tan\left(\frac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)\tan\left(\frac{\omega_c}{2}\right)$ $\hat{\omega}_{c2},\hat{\omega}_{c1}=$ desired upper and lower cutoff frequencies

Tables of Common Transform Pairs

2012 by Marc Ph. Stoecklin — marc@stoecklin.net — http://www.stoecklin.net/ — 2012-12-20 — version v1.5.3

Engineers and students in communications and mathematics are confronted with transformations such as the z-Transform, the Fourier transform, or the Laplace transform. Often it is quite hard to quickly find the appropriate transform in a book or the Internet, much less to have a comprehensive overview of transformation pairs and corresponding properties.

In this document I compiled a handy collection of the most common transform pairs and properties of the

- ▷ **continuous-time frequency Fourier transform** $(2\pi f)$,
- ▷ **continuous-time pulsation Fourier transform** (ω),
- ▷ **z-Transform**,
- ▷ **discrete-time Fourier transform DTFT**, and
- ▷ **Laplace transform**.

Please note that, before including a transformation pair in the table, I verified its correctness. Nevertheless, it is still possible that you may find errors or typos. I am very grateful to everyone dropping me a line and pointing out any concerns or typos.

Notation, Conventions, and Useful Formulas

Imaginary unit	$j^2 = -1$
Complex conjugate	$z = a + jb \quad \longleftrightarrow \quad z^* = a - jb$
Real part	$\mathfrak{Re}\left\{f(t)\right\} = \frac{1}{2}\left[f(t) + f^*(t)\right]$
Imaginary part	$\mathfrak{Im}\left\{f(t)\right\} = \frac{1}{2j}\left[f(t) - f^*(t)\right]$
Dirac delta/Unit impulse	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$
Heaviside step/Unit step	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$
Sine/Cosine	$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
Sinc function	$\operatorname{sinc}(x) \equiv \frac{\sin(x)}{x}$ (unnormalized)
Rectangular function	$\operatorname{rect}(\frac{t}{T}) = \begin{cases} 1 & \text{if } \lvert t\rvert \leqslant \frac{T}{2} \\ 0 & \text{if } \lvert t\rvert > \frac{T}{2} \end{cases}$
Triangular function	$\operatorname{triang}\left(\frac{t}{T}\right) = \operatorname{rect}(\frac{t}{T}) * \operatorname{rect}(\frac{t}{T}) = \begin{cases} 1 - \frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T \\ 0 & \lvert t\rvert > T \end{cases}$
Convolution	continuous-time: $(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)\, g^*(t-\tau) d\tau$
	discrete-time: $(u * v)[n] = \sum_{m=-\infty}^{\infty} u[m]\, v^*[n-m]$
Parseval theorem	general statement: $\int_{-\infty}^{+\infty} f(t) g^*(t) dt = \int_{-\infty}^{+\infty} F(f) G^*(f) df$
	continuous-time: $\int_{-\infty}^{+\infty} \lvert f(t)\rvert^2 dt = \int_{-\infty}^{+\infty} \lvert F(f)\rvert^2 df$
	discrete-time: $\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$
Geometric series	$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad \sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$
	in general: $\sum_{k=m}^{n} x^k = \frac{x^m - x^{n+1}}{1-x}$

Table of Continuous-time Frequency Fourier Transform Pairs

	$f(t) = \mathcal{F}^{-1}\{F(f)\} = \int_{-\infty}^{+\infty} f(t) e^{j2\pi ft} df$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t) e^{-j2\pi ft} dt$	
transform	$f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$	
time reversal	$f(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(-f)$	frequency reversal
complex conjugation	$f^*(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F^*(-f)$	reversed conjugation
reversed conjugation	$f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F^*(f)$	complex conjugation
	$f(t)$ is purely real	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = F^*(-f)$	even/symmetry
	$f(t)$ is purely imaginary	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = -F^*(-f)$	odd/antisymmetry
even/symmetry	$f(t) = f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$ is purely real	
odd/antisymmetry	$f(t) = -f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$ is purely imaginary	
time shifting	$f(t-t_0)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)e^{-j2\pi f t_0}$	
	$f(t)e^{j2\pi f_0 t}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f-f_0)$	frequency shifting
time scaling	$f(af)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert} F\left(\frac{f}{a}\right)$	
	$\frac{1}{\lvert a\rvert} f\left(\frac{f}{a}\right)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(af)$	frequency scaling
linearity	$af(t) + bg(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$aF(f) + bG(t)$	
time multiplication	$f(t)g(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) * G(f)$	frequency convolution
frequency convolution	$f(t) * g(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)G(f)$	frequency multiplication
delta function	$\delta(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-t_0)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{-j2\pi f t_0}$	
	1	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\delta(f)$	delta function
	$e^{j2\pi f_0 t}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\delta(f-f_0)$	shifted delta function
two-sided exponential decay	$e^{-a\lvert t\rvert} \quad a > 0$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{2a}{a^2+4\pi^2 f^2}$	
	$e^{-\pi t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{-\pi f^2}$	
	$e^{j\pi t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{j\pi\left(\frac{1}{4}-f^2\right)}$	
sine	$\sin(2\pi f_0 t + \phi)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{j}{2}\left[e^{-j\phi}\delta(f+f_0) - e^{j\phi}\delta(f-f_0)\right]$	
cosine	$\cos(2\pi f_0 t + \phi)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{2}\left[e^{-j\phi}\delta(f+f_0) + e^{j\phi}\delta(f-f_0)\right]$	
sine modulation	$f(t)\sin(2\pi f_0 t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{j}{2}\left[F(f+f_0) - F(f-f_0)\right]$	
cosine modulation	$f(t)\cos(2\pi f_0 t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{2}\left[F(f+f_0) + F(f-f_0)\right]$	
squared sine	$\sin^2(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{4}\left[2\delta(f) - \delta\left(f-\frac{1}{\pi}\right) - \delta\left(f+\frac{1}{\pi}\right)\right]$	
squared cosine	$\cos^2(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{4}\left[2\delta(f) + \delta\left(f-\frac{1}{\pi}\right) + \delta\left(f+\frac{1}{\pi}\right)\right]$	
rectangular	$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \lvert t\rvert \leqslant \frac{T}{2} \\ 0 & \lvert t\rvert > \frac{T}{2} \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$T \,\mathrm{sinc}\, Tf$	
triangular	$\mathrm{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T \\ 0 & \lvert t\rvert > T \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$T \,\mathrm{sinc}^2\, Tf$	
step	$u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{j2\pi f} + \delta(f)$	
signum	$\mathrm{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{j\pi f}$	
sinc	$\mathrm{sinc}(Bt)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{B}\mathrm{rect}\left(\frac{f}{B}\right) = \frac{1}{B} 1_{\left[-\frac{B}{2},+\frac{B}{2}\right]}(f)$	
squared sinc	$\mathrm{sinc}^2(Bt)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{B}\mathrm{triang}\left(\frac{f}{B}\right)$	
n-th time derivative	$\frac{d^n}{dt^n} f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$(j2\pi f)^n F(f)$	
n-th frequency derivative	$t^n f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{(-j2\pi)^n} \frac{d^n}{df^n} F(f)$	
	$\frac{1}{1+t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\pi e^{-2\pi\lvert f\rvert}$	

Table of Continuous-time Pulsation Fourier Transform Pairs

	$x(t) = \mathcal{F}_\omega^{-1}\{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t}d\omega$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega) = \mathcal{F}_\omega\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$	
transform	$x(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$	
time reversal	$x(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(-\omega)$	frequency reversal
complex conjugation	$x^*(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(-\omega)$	reversed conjugation
reversed conjugation	$x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(\omega)$	complex conjugation
	$x(t)$ is purely real	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(f) = X^*(-\omega)$	even/symmetry
	$x(t)$ is purely imaginary	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(f) = -X^*(-\omega)$	odd/antisymmetry
even/symmetry	$x(t) = x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely real	
odd/antisymmetry	$x(t) = -x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely imaginary	
time shifting	$x(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)e^{-j\omega t_0}$	
	$x(t)e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega-\omega_0)$	frequency shifting
time scaling	$x(af)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	
	$\frac{1}{\lvert a\rvert}x\left(\frac{f}{a}\right)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(a\omega)$	frequency scaling
linearity	$ax_1(t) + bx_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$aX_1(\omega) + bX_2(\omega)$	
time multiplication	$x_1(t)x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$	frequency convolution
frequency convolution	$x_1(t) * x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X_1(\omega)X_2(\omega)$	frequency multiplication
delta function	$\delta(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$e^{-j\omega t_0}$	
	1	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega)$	delta function
	$e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega-\omega_0)$	shifted delta function
two-sided exponential decay	$e^{-a\lvert t\rvert} \quad a>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2a}{a^2+\omega^2}$	
exponential decay	$e^{-at}u(t) \quad \Re\{a\}>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a+j\omega}$	
reversed exponential decay	$e^{-at}u(-t) \quad \Re\{a\}>0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a-j\omega}$	
	$e^{\frac{t^2}{2\sigma^2}}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$	
sine	$\sin(\omega_0 t+\phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j\pi\left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right]$	
cosine	$\cos(\omega_0 t+\phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\left[e^{-j\phi}\delta(\omega+\omega_0) + e^{j\phi}\delta(\omega-\omega_0)\right]$	
sine modulation	$x(t)\sin(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]$	
cosine modulation	$x(t)\cos(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$	
squared sine	$\sin^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2[2\delta(f) - \delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$	
squared cosine	$\cos^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2[2\delta(\omega) + \delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$	
rectangular	$\text{rect}\left(\frac{t}{T}\right) = \begin{cases}1 & \lvert t\rvert \leqslant \frac{T}{2}\\ 0 & \lvert t\rvert > \frac{T}{2}\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\,\text{sinc}\left(\frac{\omega T}{2}\right)$	
triangular	$\text{triang}\left(\frac{t}{T}\right) = \begin{cases}1-\frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T\\ 0 & \lvert t\rvert > T\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\,\text{sinc}^2\left(\frac{\omega T}{2}\right)$	
step	$u(t) = 1_{[0,+\infty]}(t) = \begin{cases}1 & t\geqslant 0\\ 0 & t<0\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\delta(f) + \frac{1}{j\omega}$	
signum	$\text{sgn}(t) = \begin{cases}1 & t\geqslant 0\\ -1 & t<0\end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2}{j\omega}$	
sinc	$\text{sinc}(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\,\text{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T,+\pi T]}(f)$	
squared sinc	$\text{sinc}^2(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\,\text{triang}\left(\frac{\omega}{2\pi T}\right)$	
n-th time derivative	$\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$(j\omega)^n X(\omega)$	
n-th frequency derivative	$t^n f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j^n\frac{d^n}{df^n}X(\omega)$	
time inverse	$\frac{1}{t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$-j\pi\text{sgn}(\omega)$	

Table of z-Transform Pairs

	$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$		$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	ROC
transform	$x[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X(z)$	R_x
time reversal	$x[-n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X(\frac{1}{z})$	$\frac{1}{R_x}$
complex conjugation	$x^*[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X^*(z^*)$	R_x
reversed conjugation	$x^*[-n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
real part	$\mathfrak{Re}\{x[n]\}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
imaginary part	$\mathfrak{Im}\{x[n]\}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting	$x[n-n_0]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$z^{-n_0}X(z)$	R_x
scaling in $\mathcal{Z}$	$a^n x[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X\left(\frac{z}{a}\right)$	$\|a\|R_x$
downsampling by N	$x[Nn], N \in \mathbb{N}_0$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{N}\sum_{k=0}^{N-1} X\left(W_N^k z^{\frac{1}{N}}\right) \quad W_N = e^{-\frac{j2\omega}{N}}$	R_x
linearity	$ax_1[n] + bx_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
time multiplication	$x_1[n]x_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{2\pi j}\oint X_1(u)X_2\left(\frac{z}{u}\right)u^{-1}du$	$R_x \cap R_y$
frequency convolution	$x_1[n] * x_2[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$X_1(z)X_2(t)$	$R_x \cap R_y$
delta function	$\delta[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	1	$\forall\, z$
shifted delta function	$\delta[n-n_0]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	z^{-n_0}	$\forall\, z$
step	$u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-1}$	$\|z\| > 1$
	$-u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-1}$	$\|z\| < 1$
ramp	$nu[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
	$n^2u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
	$-n^2u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\|z\| < 1$
	$n^3u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\|z\| > 1$
	$-n^3u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\|z\| < 1$
	$(-1)^n$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z+1}$	$\|z\| < 1$
exponential	$a^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
	$-a^n u[-n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-a}$	$\|z\| < \|a\|$
	$a^{n-1}u[n-1]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1}{z-a}$	$\|z\| > \|a\|$
	$na^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
	$n^2a^n u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{az(z+a}{(z-a)^3}$	$\|z\| > \|a\|$
	$e^{-an}u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{z-e^{-a}}$	$\|z\| > \|e^{-a}\|$
exp. interval	$\begin{cases} a^n & n = 0,\ldots,N-1 \\ 0 & \text{otherwise} \end{cases}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$
sine	$\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z\sin(\omega_0)}{z^2-2\cos(\omega_0)z+1}$	$\|z\| > 1$
cosine	$\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z-\cos(\omega_0))}{z^2-2\cos(\omega_0)z+1}$	$\|z\| > 1$
	$a^n\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{za\sin(\omega_0)}{z^2-2a\cos(\omega_0)z+a^2}$	$\|z\| > a$
	$a^n\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z(z-a\cos(\omega_0))}{z^2-2a\cos(\omega_0)z+a^2}$	$\|z\| > a$
differentiation in $\mathcal{Z}$	$nx[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$-z\frac{dX(z)}{dz}$	R_x
integration in $\mathcal{Z}$	$\frac{x[n]}{n}$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$-\int_0^z \frac{X(z)}{z}dz$	R_x
	$\frac{\prod_{i=1}^{m}(n-i+1)}{a^m m!}a^m u[n]$	$\overset{\mathcal{Z}}{\Longleftrightarrow}$	$\frac{z}{(z-a)^{m+1}}$	

Note:

$$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

Table of Common Discrete Time Fourier Transform (DTFT) Pairs

	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$	
transform	$x[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$	
time reversal	$x[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{-j\omega})$	
complex conjugation	$x^*[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X^*(e^{-j\omega})$	
reversed conjugation	$x^*[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X^*(e^{j\omega})$	
	$x[n]$ is purely real	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = X^*(e^{-j\omega})$	even/symmetry
	$x[n]$ is purely imaginary	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$	odd/antisymmetry
even/symmetry	$x[n] = x^*[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$ is purely real	
odd/antisymmetry	$x[n] = -x^*[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$ is purely imaginary	
time shifting	$x[n-n_0]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})e^{-j\omega n_0}$	
	$x[n]e^{j\omega_0 n}$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j(\omega-\omega_0)})$	frequency shifting
downsampling by N	$x[Nn] \quad N \in \mathbb{N}_0$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{N}\sum_{k=0}^{N-1} X(e^{j\frac{\omega-2\pi k}{N}})$	
upsampling by N	$\begin{cases} x\left[\frac{n}{N}\right] & n = kN \\ 0 & otherwise \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{jN\omega})$	
linearity	$ax_1[n] + bx_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$	
time multiplication	$x_1[n]x_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X_1(e^{j\omega}) * X_2(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X_1(e^{j(\omega-\sigma)})X_2(e^{j\sigma})d\sigma$	frequency convolution
frequency convolution	$x_1[n] * x_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X_1(e^{j\omega})X_2(e^{j\omega})$	frequency multiplication
delta function	$\delta[n]$	$\overset{DTFT}{\Longleftrightarrow}$	1	
shifted delta function	$\delta[n-n_0]$	$\overset{DTFT}{\Longleftrightarrow}$	$e^{-j\omega n_0}$	
	1	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\delta}(\omega)$	delta function
	$e^{j\omega_0 n}$	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\delta}(\omega-\omega_0)$	shifted delta function
sine	$\sin(\omega_0 n + \phi)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{j}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) - e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$	
cosine	$\cos(\omega_0 n + \phi)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) + e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$	
rectangular	$\text{rect}\left(\frac{n}{M}\right) = \begin{cases} 1 & \lvert n\rvert \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega(M+\frac{1}{2})]}{\sin(\omega/2)}$	
step	$u[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-e^{-j\omega}} + \frac{1}{2}\tilde{\delta}(\omega)$	
decaying step	$a^n u[n] \quad (\lvert a\rvert < 1)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-ae^{-j\omega}}$	
special decaying step	$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{(1-ae^{-j\omega})^2}$	
sinc	$\frac{\sin(\omega_c n)}{\pi n} = \frac{\omega_c}{\pi}\,\text{sinc}(\omega_c n)$	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\text{rect}}\left(\frac{\omega}{\omega_c}\right) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert < \pi \end{cases}$	
MA	$\text{rect}\left(\frac{n}{M} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$	
MA	$\text{rect}\left(\frac{n}{M-1} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M-1 \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega M/2]}{\sin(\omega/2)}e^{-j\omega(M-1)/2}$	
derivation	$nx[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$j\frac{d}{d\omega}X(e^{j\omega})$	
difference	$x[n] - x[n-1]$	$\overset{DTFT}{\Longleftrightarrow}$	$(1-e^{-j\omega})X(e^{j\omega})$	
	$\frac{a^n \sin[\omega_0(n+1)]}{\sin\omega_0}u[n] \quad \lvert a\rvert < 1$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-2a\cos(\omega_0 e^{-j\omega}) + a^2 e^{-j2\omega}}$	

Note:

$$\tilde{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega + 2\pi k) \qquad \tilde{\text{rect}}(\omega) = \sum_{k=-\infty}^{+\infty} \text{rect}(\omega + 2\pi k)$$

Parseval:

$$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$$

Table of Laplace Transform Pairs

	$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \lim_{T\to\infty} \int_{c-jT}^{c+jT} F(s)e^{st}ds$		$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$	
transform	$f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s)$	
complex conjugation	$f^*(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F^*(s^*)$	
time shifting	$f(t-a) \quad t \geqslant a > 0$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$a^{-as}F(s)$	
	$e^{-at}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s+a)$	frequency shifting
time scaling	$f(at)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}F(\frac{s}{a})$	
linearity	$af_1(t) + bf_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$aF_1(s) + bF_2(s)$	
time multiplication	$f_1(t)f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s) * F_2(s)$	frequency convolution
time convolution	$f_1(t) * f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s)F_2(s)$	frequency product
delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}	exponential decay
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$	
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$	
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$	
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$	
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$	
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$	
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$	
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$	
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$	
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$	
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$	
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$	
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$	
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$	
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$	
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$	
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$	
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s) - f(0)$	
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s) - sf(0) - f'(0)$	
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$	
time integration	$\int_0^t f(\tau)d\tau = (u * f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$	
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$	
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$	
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$	